3rd Lecture of Operation Research 2

Relation between Primal and Dual problems solution at any iteration:

1st Method:

(Objective Coefficient Of a variable X .. c X) = L.H.S – R.H.S of the corresponding constraint.

Objective Coefficient Of a variable:

أختار أى Variables من اللى موجودين فى الـ Objective fun. بس من الافضل إنك تختار الـ Starting basic variables وبعد كده تأخد الـ Coeff. بتوعهم اللى موجودين فى صف الـ Z فى الـ iteration اللى انت شغال عليها بعد كده تشوف

الـ Corresponding constraint للـ Variables اللى انت أختارتها وتطرح الـ L.H.S من الـ R.H.S وتساويه بالـ Coeff. اللى انت جيبته .

(Objective Coefficient Of a variable J .. c J) = L.H.S – R.H.S of the corresponding constraint.

Basic	X1	X2	X3	S1	R	Sol.
Z	0	0	3/5	29/5	-2/5+M	54 4/5
X2	0	1	-1/5	2/5	-1/5	12/5
X1	1	0	7/5	1/5	2/5	26/5

S1: $Y1 \geq 0$ $29/5 = Y1 - 0$ $Y1_1^* = 29/5$

R: $Y2 \geq -M$ $-2/5 + M = Y2 + M$ $Y_2^* = -2/5$

$$Z^* = W^* = 54\ \frac{4}{5}$$

2nd Method:

(Variables Values) = (Corresponding basic variable coefficient in the same order)[Inverse Matrix]

Corresponding basic variable coefficient in the same order:

معاملات الـ Basic Variables اللى موجودين فى الـ Iteration اللى أنا بجيب عندها العلاقة بين الـ Primal Solution و الـ Dual Solution بنفس الترتيب اللى موجودين بيه فى الجدول فى عمود الـ Basic وبجيبهم من الـ Original Problem

Basic	X1	X2	X3	S1	R	Sol.
Z	0	0	3/5	29/5	-2/5+M	54 4/5
X2	0	1	-1/5	2/5	-1/5	12/5
X1	1	0	7/5	1/5	2/5	26/5

لو مثلا الـ iteration اللى فوق دى هى اللى انا شغال عليها يبقى الـ Basic Variables اللى عندى هما X1 , X2 بنفس الترتيب يعنى X2 الاول وبعدين X1 طب كده انا عرفت الـ Variables عايز اجيب الـ Coefficient بتعوتهم هجيبهم منين من الـ Objective fun.

اللى عندى فى المسأله $\text{Max } Z = 5 X1 + 12 X2 + 4 X3 + 0 S1 + 0 R$

يبقى معامل الـ X2 بـ 12 ومعامل الـ X1 بـ 5 يبقى الـ Row Matrix هتبقى (12 5)

Inverse Matrix:

The Matrix under the Starting basic variables

Basic	X1	X2	X3	S1	R	Sol.
Z	0	0	3/5	29/5	-2/5+M	54 4/5
X2	0	1	-1/5	2/5	-1/5	12/5
X1	1	0	7/5	1/5	2/5	26/5

الـ Starting basic variables فى اول iteration كانوا S1 , R يبقى الـ IM او الـ Inverse Matrix هى الـ Matrix اللى تحت

الـ S1 و R اللى هما ملونين باللون الاصفر فى الـ Iteration اللى فوق.

$$IM = \begin{pmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{pmatrix}$$

(Variables Values) = (Corresponding basic variable coefficient in the same order)[Inverse Matrix]

$$(Y1 \quad Y2) = (12 \quad 5)\begin{pmatrix} 2/5 & -1/5 \\ 1/5 & 2/5 \end{pmatrix} = (29/5 \quad -2/5)$$

$$Y1_1^* = 29/5 \qquad Y_2^* = -2/5 \qquad Z^* = W^* = 54\,\tfrac{4}{5}$$

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Max	$\longrightarrow$	Objective value	$\longleftarrow$	Min
W		$Z^* = W^*$		Z

Any objective feasible solution of Max Prob. $\leq$ Any objective feasible solution of Min Prob.

Equality condition occurred at optimal solution.

علشان اقارن بين اتنين Solution لازم يحققوا شرط الـ feasibility اولا علشان اقدر اقارن بينهم يعنى لازم يكونوا بيحققوا كل الـ Constraints اللى موجودة.

Example no. 1:

Estimate a range for the optimal objective value for the linear programming problem:

$$\text{Min } Z = 5X_1 + 2X_2$$

S.T:

$$X_1 - X_2 \geq 3$$

$$2X_1 + 3X_2 \geq 5$$

$$X_1 , X_2 \geq 0$$

Then determine whether the following pairs of primal and dual solutions are optimal .

A- ($X_1 = 3$, $X_2 = 1$, $Y_1 = 4$, $Y_2 = 1$)
B- ($X_1 = 4$, $X_2 = 1$, $Y_1 = 1$, $Y_2 = 0$)
C- ($X_1 = 3$, $X_2 = 0$, $Y_1 = 5$, $Y_2 = 0$)

Solution

Standard Form for Primal Prob:

$$\text{Min } Z = 5X_1 + 2X_2 - 0S_1 - 0S_2$$

S.T:

$$X_1 - X_2 - S_1 = 3$$

$$2X_1 + 3X_2 - S_2 = 5$$

$$X_1 , X_2 \geq 0$$

Dual:

$$\text{Max } W = 3Y_1 + 5Y_2$$

S.T:

$$Y_1 + 2Y_2 \leq 5$$

$$-Y_1 + 3Y_2 \leq 2$$

$$Y_1 \geq 0$$

$$Y_2 \geq 0$$

The Optimal Solution is located between optimal primal and optimal dual

Let X1 = 4 , X2 = 1

Then sub in Constraints to check feasibility

$X1 - X2 \geq 3$

$4 - 1 \geq 3$ √

$2 X1 + 3 X2 \geq 5$

$8 + 3 \geq 5$ √

To get the upper bound sub in Min objective fun

Min $Z = 5 X1 + 2 X2$

$Z = 5*4 + 2*1 = 22$

Let Y1 = 2 , Y2 = 1

Then sub in Constraints to check feasibility

$Y1 + 2 Y2 \leq 5$

$- Y1 + 3 Y2 \leq 2$

$Y1 \geq 0$

$Y2 \geq 0$

To get the lower bound sub in Max objective fun

Max $W = 3 Y1 + 5 Y2$

$W = 3*2 + 5*1 = 11$

$W^* \leq 22$ ، $Z^* \geq 11$

Then determine whether the following pairs of primal and dual solutions are optimal .

A- (X1= 3 , X2 = 1 , Y1 = 4 , Y2 =1)

Sub in Constraints to check feasibility

$X1 - X2 \geq 3$

$3 - 1 = 2 \ngeq 3$ It is not feasible solution, Then this is not optimal solution.

B- ($X1 = 4$, $X2 = 1$, $Y1 = 1$, $Y2 = 0$)

Sub in Constraints to check feasibility

$X1 - X2 = 3 \geq 3$ √

$2\,X1 + 3\,X2 = 8 + 3 = 11 \geq 5$ √

$Y1 + 2\,Y2 = 1 + 0 = 1 \leq 5$ √

$-\,Y1 + 3\,Y2 = -1 + 0 = -1 \leq 2$ √

Then it is feasible solution.

Sub in objective fun to check optimality

$Z = 5\,X1 + 2\,X2 = 22$

$W = 3\,Y1 + 5\,Y2 = 3$

$Z \neq W$ Then it is not optimal solution.

C- ($X1 = 3$, $X2 = 0$, $Y1 = 5$, $Y2 = 0$)

Sub in Constraints to check feasibility

$X1 - X2 = 3 \geq 3$ √

$2\,X1 + 3\,X2 = 6 + 0 = 6 \geq 5$ √

$Y1 + 2\,Y2 = 5 + 0 = 5 \leq 5$ √

$-\,Y1 + 3\,Y2 = -5 + 0 = -5 \leq 2$ √

Then it is feasible solution.

Sub in objective fun to check optimality

$Z = 5\,X1 + 2\,X2 = 15$

$W = 3\,Y1 + 5\,Y2 = 15$

$Z = W$ Then it is optimal solution.

Best Wishes